Satellites of Dipole-Forbidden and Dipole-Allowed Spectral Lines Under a Strong or High-Frequency Quasimonochromatic Electric Field in Plasmas

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Abstract

We study analytically the satellites of dipole-forbidden and dipole-allowed spectral lines for the situation where in the plasma there is either a strong or high-frequency Quasimonochromatic Electric Field (QEF) of the frequency ω, representing, e.g., the Langmuir wave or an optical laser (or microwave) radiation penetrating the plasma. We demonstrate that the resulting sequence of satellites of the dipole-forbidden and dipole-allowed lines is such that the separation between any two adjacent satellites is 2ω. We show that the separation of any satellite from the unperturbed position of the dipole-forbidden or dipole allowed line is *not* an integer number of ω – all of them are *shifted* by the same amount controlled by the parameters of the QEF and of the radiating atom/ion. Based on this shift, we propose a new method for the spectroscopic diagnostic of the parameters of the QEF in plasmas – the method having an advantage over the existing methods. We also provide some examples of spectral lines of atoms and molecules that can be employed for taking the advantage of this diagnostic method.

Key words: satellites of spectral lines; quasimonochromatic electric field; Langmuir waves; laser radiation; shift of satellites; spectroscopic diagnostic method

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1. Introduction

The satellites of dipole-forbidden and dipole-allowed spectral lines are possible, e.g., in spectra of helium and lithium atoms, as well as in spectra of the He-like and Li-like ions (though not only). For instance, Baranger and Mozer [1] considered the situation where in the energy spectrum of a helium atom one can select the system of three levels 0, 1, and 2, having the following properties. Level 2 is coupled by dipole matrix elements with a closely located level 1 (the separation being denoted as Δ) and to the distant level 0, while the levels 1 and 0 are not coupled by a dipole matrix element. So, at the absence of an external electric field, the radiative transition from 2 to 0 is allowed, while the radiative transition from 1 to 0 is forbidden (in the dipole approximation). Under a static electric field **F**, in addition to the allowed spectral line at the frequency $\omega_{20} = \omega_2 - \omega_0$, the dipole-forbidden spectral line shows up at the frequency $\omega_{10} = \omega_1 - \omega_0$ (see Fig. 1). We use atomic units, so that the frequencies ω_0, ω_1, ω_2 are the energies of the levels 0, 1, 2.

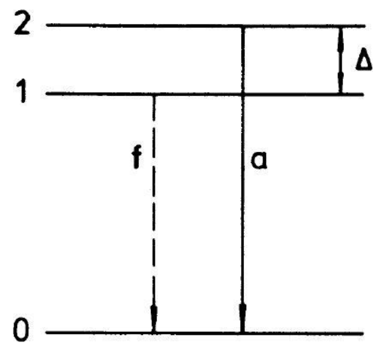

Fig. 1. The system of three levels 0, 1, and 2. The dipole-allowed and dipole-forbidden spectral lines are shown by solid and dashed arrows, respectively.

The satellites can be caused by a Quasimonochromatic Electric Field (QEF). In plasmas, the QEF could represent, e.g., the Langmuir wave or an optical laser (or microwave) radiation penetrating the plasma. Below is a brief overview of the corresponding analytical results.

Under a relatively weak QEF of the frequency ω, two satellites could show up at the frequencies $\omega_{sat} = \omega_{10} \pm \omega$, instead of the dipole-forbidden spectral line. For the case of the isotropic multimode QEF, Baranger and Mozer [1] used the standard nonstationary perturbation theory to obtain the ratio of the satellites intensities S_+ and S_- to the intensity I_a of the allowed spectral line and to propose the corresponding method for plasma diagnostics.

Several years later, Cooper and Ringler [2] performed similar calculations in frames of the standard nonstationary perturbation theory for the case where the QEF has a single mode and is linearly-polarized, being in the form of

$$\mathbf{E}(t) = \mathbf{E}_0 \cos \omega t. \tag{1}$$

Baranger and Mozer suggested measuring the QEF amplitude by measuring experimental ratio $S_\pm/I_a$ and then comparing it with the correspondent theoretical ratio. This idea was then employed by various authors in several experiments. However, soon the experiments entered the situation where the QEF was relatively strong, so that the Baranger-Mozer theoretical results [1] and the Cooper-Ringler theoretical results [2] became invalid.

A number of these later experiments related to the low-frequency case, where $\omega << \Delta$. In this case, the employment of the adiabatic perturbation theory allowed obtaining the corresponding analytical results beyond the limits of validity of the standard nonstationary perturbation theory – see paper [3] and Sect. 5.1 of book [4]. For illustrating this, we denote:

$$\alpha = 2E_0 z_{12}/\Delta. \tag{2}$$

The nonstationary perturbation theory is valid only for $\alpha << 1$. In distinction, the adiabatic perturbation theory is valid for

$$\alpha\omega/\Delta << 1. \tag{3}$$

So, as long as $\omega << \Delta$, the condition (3) can be satisfied even for $\alpha > 1$, that is, for the stronger QEF. The analytical results, obtained by using the adiabatic perturbation theory allowed the experimental determination of the QEF amplitude for the relatively strong fields – see, e.g., Sect. 7.6.2 of book [4].

In the present paper we study these satellites analytically for the situation where in the plasma there is either a strong or high-frequency QEF (so that this includes the scenario opposite to the one underlying the adiabatic theory of the satellites). We demonstrate that the resulting sequence of satellites of the dipole-forbidden and dipole-allowed lines is such that the separation between any two adjacent satellites is 2ω. We show analytically that the separation of any satellite from the unperturbed position of the dipole-forbidden or dipole allowed line is *not* an integer number of ω – all of them are *shifted* by the same amount controlled by the parameters of the QEF and of the radiating atom/ion. Based on this shift, we propose a new method for the spectroscopic diagnostic of the parameters of the QEF in plasmas – the method having an advantage over the existing methods.

2. Analytical results and a new method for spectroscopic diagnostic of plasmas

We analyze the system of levels shown in Fig. 1 under the linearly-polarized field $\mathbf{E}(t) = \mathbf{E}_0 \cos\omega t$ that is either strong or high-frequency (or both), as manifested by the small parameter

$$\delta = \Delta^2/\max(z_{12}E_0\omega, \omega^2) << 1. \qquad (4)$$

In Eq. (4), the corresponding matrix element of the z-coordinate, chosen parallel to the field amplitude $\mathbf{E}_0$, is denoted z_{12}.

After neglecting the terms that rapidly oscillate, the solution for the probability amplitudes $C_1(t)$ and $C_2(t)$ is [5, 6]

$$C_{1,2}(t) = 2^{-3/2}\exp[-i(\omega_1 + \omega_2)t/2]\{\cos(Qt) \exp[-(iA)\sin\omega t] \pm \sin(Qt) \exp[(iA)\sin\omega t]\}. \qquad (5)$$

In Eq. (5) it was denoted

$$Q = J_0(2A)\Delta/2, \qquad A = z_{12}E_0/\omega. \qquad (6)$$

In Eq. (6), $J_0(2A)$ is the Bessel function. The perturbed wave function is

$$\psi(t) = C_1(t)\varphi_1 + C_2(t)\varphi_2. \qquad (7)$$

where φ_1 and φ_2 are the unperturbed wave functions.

The quasienergy of the perturbed level 1 is by $[1 - J_0(2A)]\Delta/2$ higher than the unperturbed energy of this level ω_1, while the quasienergy of the perturbed level 2 is by $[1 - J_0(2A)]\Delta/2$ lower than the unperturbed energy of this level ω_2. Thus, in some sense, the strong or high-frequency QEF causes the mutual “attraction” of the levels 1 and 2.

In the present work, by using Eqs. (5) and (6), we calculated analytically the spectral profile of the radiative transition from the intermixed levels 1 and 2 to the level 0. For the spectral profile “in the vicinity” of the dipole-forbidden line, we obtained:

k=∞

$$S_f(\Delta\omega) = \sum_{k=-\infty} J_{2k-1}{}^2(A)\ \delta\{\Delta\omega - [1 - J_0(2A)]\Delta/2 - (2k\text{-}1)\omega\}. \tag{8}$$

For the spectral profile "in the vicinity" of the dipole-allowed line, we obtained:

$$S_a(\Delta\omega) = \sum_{k=-\infty}^{k=\infty} J_{2k-1}{}^2(A)\ \delta\{\Delta\omega + [1 - J_0(2A)]\Delta/2 - 2k\omega\}. \tag{9}$$

Figure 2 shows the dependence of the scaled profile $S_f(\delta\omega)$, where $\delta\omega = \Delta\omega/\omega$, in the vicinity of the dipole-forbidden transition on the scaled QEF amplitude A (defined in Eq. (6)) for Δ/ω = 0.02. Here and below, all frequencies are measured in units of the QEF frequency ω, unless specified to the contrary; to each satellite there is assigned the Lorentzian shape of the Half Width at Half Maximum (HWHM) equal to 1/4.

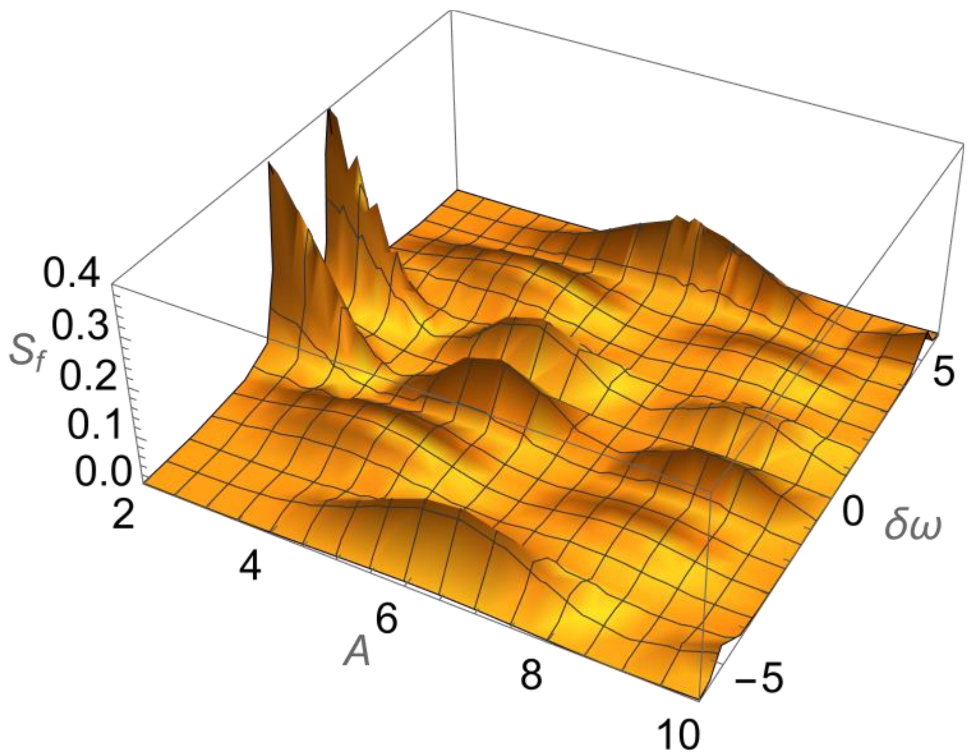

Fig. 2. Dependence of the scaled profile $S_f(\delta\omega)$, where $\delta\omega = \Delta\omega/\omega$, in the vicinity of the dipole-forbidden transition on the scaled QEF amplitude A (defined in Eq. (6)) for Δ/ω = 0.02. In this and other figures, all frequencies are measured in units of the QEF frequency ω, unless specified to the contrary. To each satellite there is assigned the Lorentzian shape of the Half Width at Half Maximum (HWHM) equal to 1/4.

Figure 3 shows the same as in Fig. 2, but for Δ/ω = 0.5.

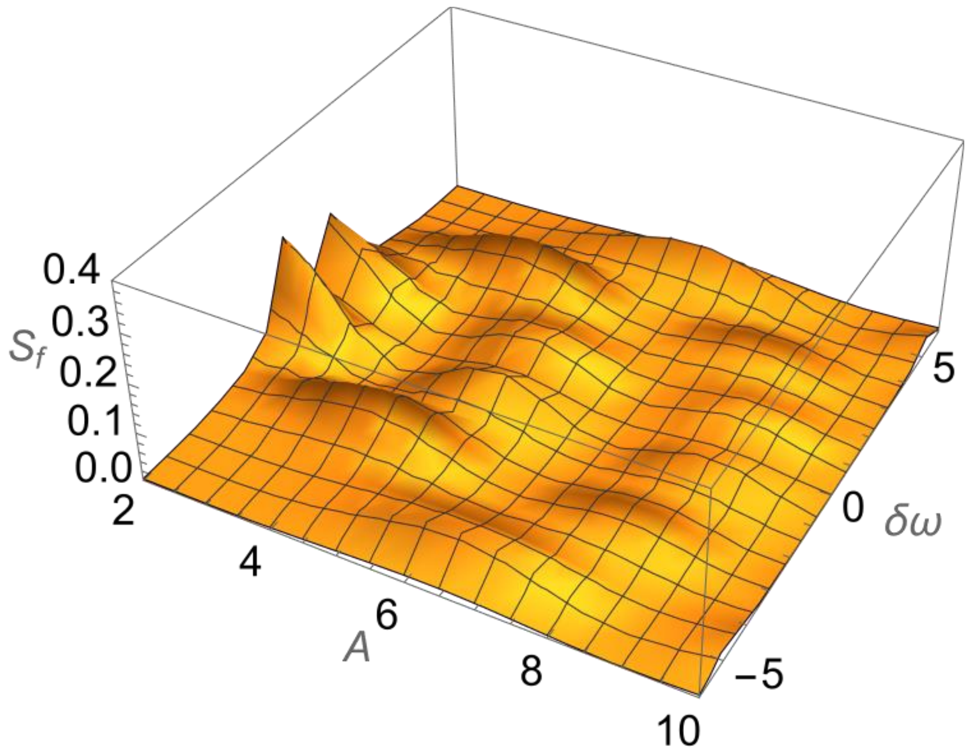

Fig. 3. The same as in Fig. 2, but for $\Delta/\omega = 0.5$.

Figure 4 shows the dependence of the scaled profile $S_a(\delta\omega)$ in the vicinity of the dipole-allowed transition on the scaled QEF amplitude A for $\Delta/\omega = 0.02$.

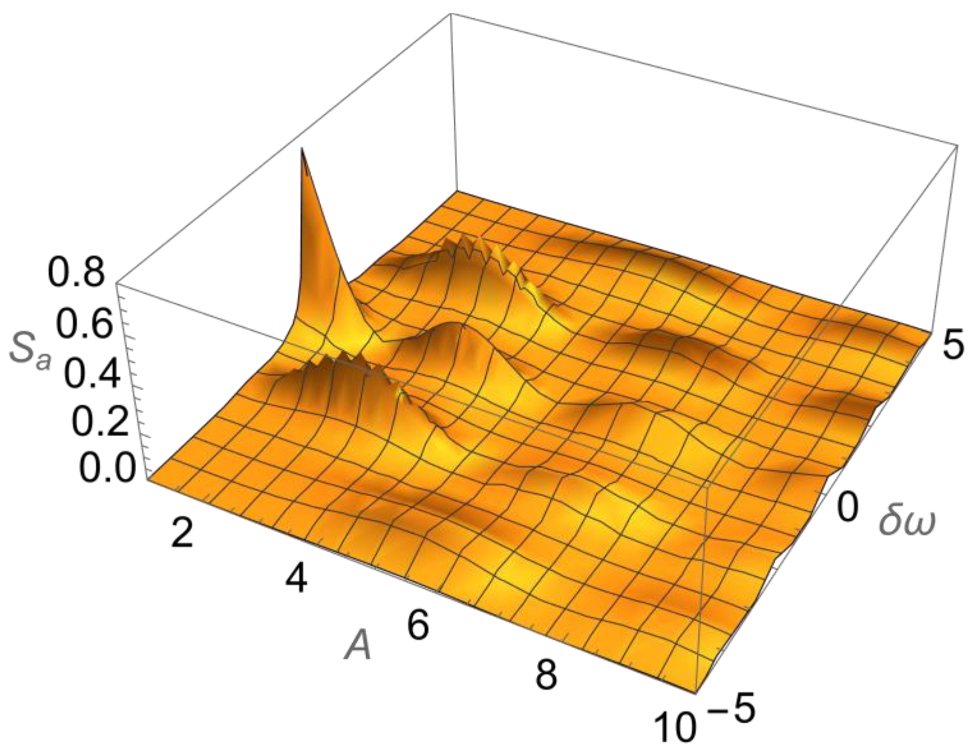

Fig. 4. Dependence of the scaled profile $S_a(\delta\omega)$ in the vicinity of the dipole-allowed transition on the scaled QEF amplitude A for $\Delta/\omega = 0.02$.

Figure 5 shows the same as in Fig. 4, but for $\Delta/\omega = 0.5$.

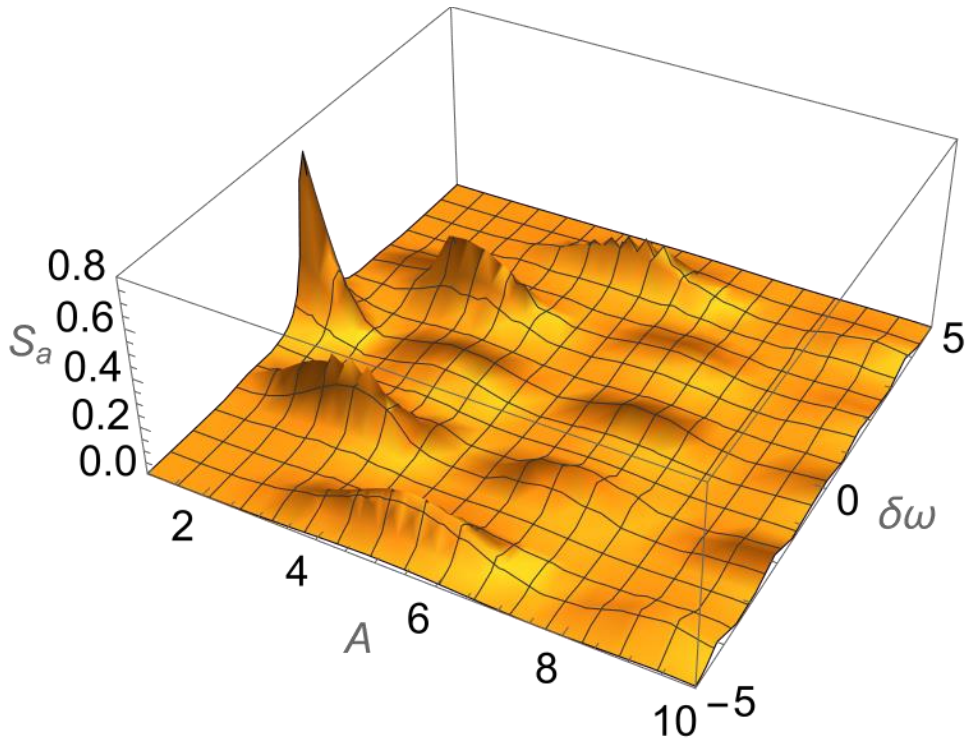

Fig. 5. The same as in Fig. 4, but for $\Delta/\omega = 0.5$.

Figure 6 presents the scaled profile $S(\delta\omega)$, combining the satellites of both the forbidden and allowed lines, versus the scaled QEF amplitude A for $\Delta/\omega = 0.02$.

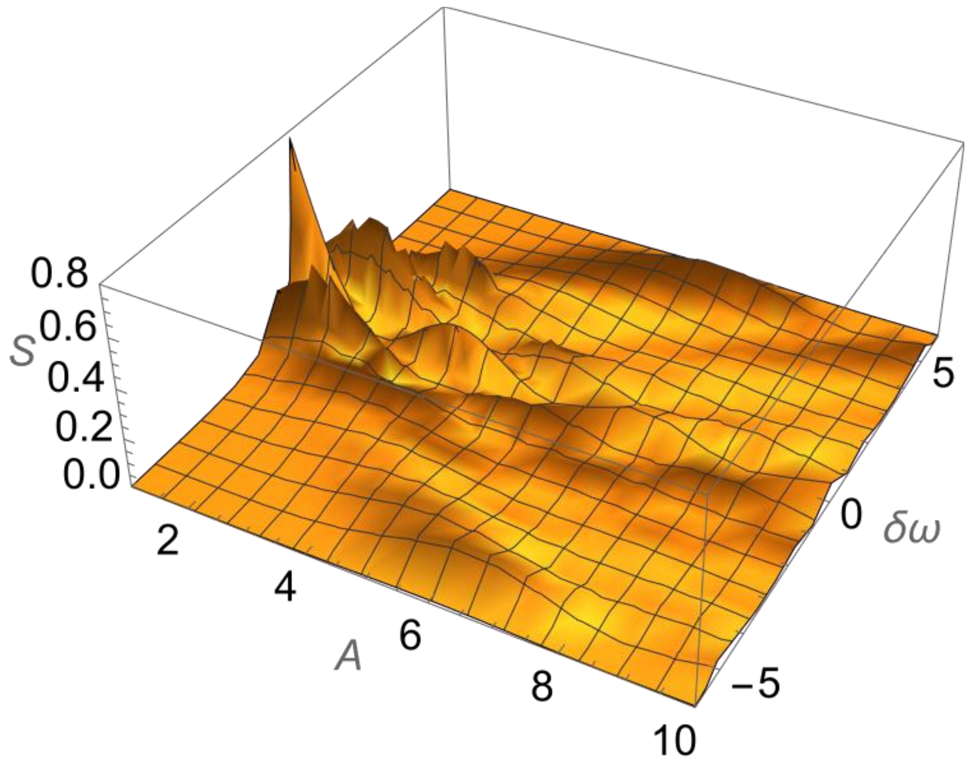

Fig. 6. The scaled profile S(δω), combining the satellites of both the forbidden and allowed lines, versus the scaled QEF amplitude A for $\Delta/\omega = 0.02$.

Figure 7 shows the same as in Fig. 6, but for $\Delta/\omega = 0.5$.

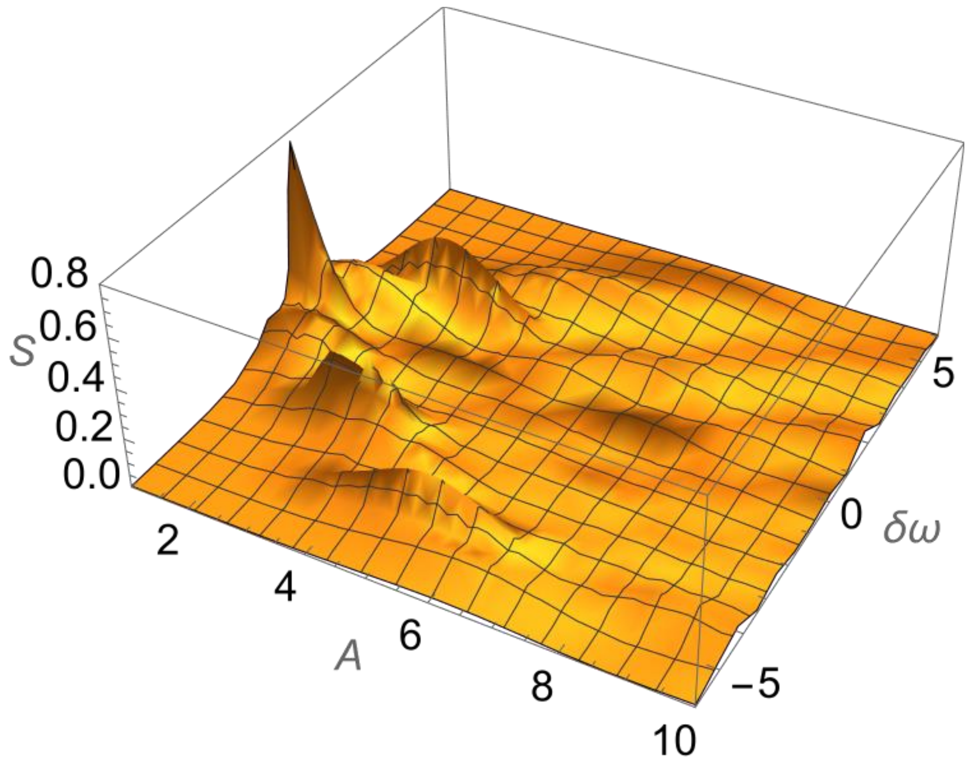

Fig. 7. The same as in Fig. 6, but for $\Delta/\omega = 0.5$.

Now we proceed to presenting a new method for spectroscopic diagnostic of plasmas – the method based on the above results. The standard method for determining the QEF amplitude E_0 is based on measuring the experimental ratio of the intensity of the first satellite in the vicinity of the forbidden line I_{s1} to the intensity of the allowed line I_a. However, in many experimental situations, the satellites are emitted from a relatively small volume (where the QEF is located), while the allowed line is emitted from a significantly larger volume. Even if there is no such form-factor, there can be a situation where the allowed line is affected by the self-absorption, while the satellites (being of significantly smaller intensity than the allowed line) are not affected. In these situations, it would be useless trying to deduce E_0 from the experimental ratio I_{s1}/I_a.

Therefore, we suggest determining the QEF amplitude E_0 from the experimental shift D of any satellite, where (according to Eqs. (8) and (9))

$$D = [1 - J_0(2A)]\Delta/2 \qquad (10)$$

Figure 8 shows the ratio of the shift D to the known separation Δ between the unperturbed levels 1 and 2 versus the scaled QEF amplitude A.

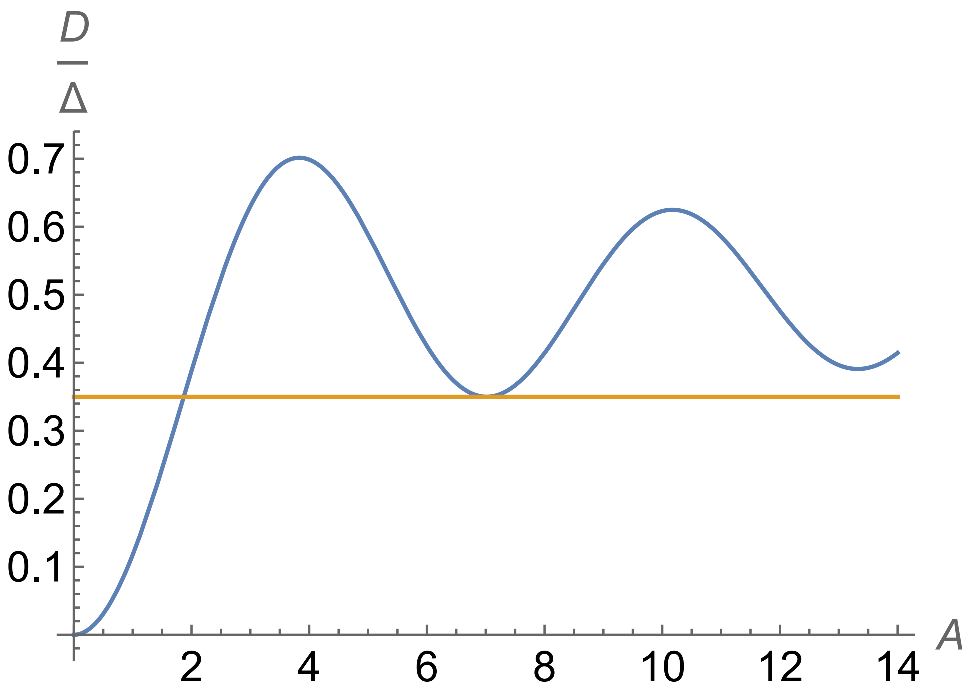

Fig. 8. Ratio of the shift D of any satellite to the known separation Δ between the unperturbed levels 1 and 2 versus the scaled QEF amplitude A(the curve). The horizontal line is at D/Δ = 0.35.

It can be seen that for D/Δ < 0.35, there is one-to-one correspondence between D/Δ and the scaled QEF amplitude A. Thus, if the experimental shift D divided by the known separation Δ is less than 0.35, then from this ratio one can determine the scaled QEF amplitude $A = z_{12}E_0/\omega$. Since the matrix element z_{12} is known and the QEF frequency ω can be found experimentally from the separation 2ω of any two adjacent satellites of either the forbidden line or the allowed line, then the QEF amplitude E_0 can be deduced from the experimentally determined value of A.

Thus, this new method allows the experimental determination of both the QEF amplitude E_0 and frequency ω in the plasma. The advantage of this new method is that it can work in the situation, where the satellites are emitted from a relatively small volume, while the allowed line is emitted from a significantly larger volume, and in the situation where the allowed line is affected by opacity.

Conclusions

We studied analytically the satellites of dipole-forbidden and dipole-allowed spectral lines for the situation where in the plasma there is either a strong or high-frequency QEF of the frequency ω. Both for the satellites of the dipole-forbidden line and for the satellites of the dipole-allowed line, the separation between any two adjacent satellites is 2ω. We demonstrated that the separation of any satellite from the unperturbed position of the dipole-forbidden or dipole allowed line is *not* an integer number of ω – all of them are *shifted* by the same amount controlled by the following two parameters: the scaled QEF amplitude $A = z_{12}E_0/\omega$ and the unperturbed separation Δ between the levels 1 and 2.

Based on the analytical expression for this shift that we derived, we proposed a new method for spectroscopic diagnostic of plasmas containing QEF. The method makes possible the experimental determination of both the QEF amplitude E_0 and frequency ω in the plasma. This method is effective even in the situations where other corresponding methods do not work. One such situation is where the satellites are emitted from a relatively small volume, while the allowed line is emitted from a significantly larger volume. Another such situation is where the allowed line is affected by opacity.

Some (out of numerous) examples of spectral lines, employment of which allows taking the advantage of this new diagnostic method, are as follows.

1. Radiative transition from the levels (3^3P, 3^3D) to 2^3S in He I; the unperturbed position of the allowed line $3^3P - 2^3S$ is at 388.9 nm.

2. Radiative transition from the levels (3^1P, 3^1D) to 2^1S in He I; the unperturbed position of the allowed line $3^1P - 2^1S$ is at 501.6 nm.

3. Radiative transition from the levels (3^2P, 3^2D) to 2^2P in Li I; the unperturbed position of the allowed line $3^2D - 2^2P$ is at 610.4 nm.

Finally, let us also provide an appropriate example of molecular spectral lines, originating from a pair of closely lying levels, whose difference in energy is due to the phenomenon of Λ-doubling of energy levels of diatomic molecules in Π-states. We remind that without the allowance for the coupling of the electronic and rotational degrees of freedom, each level of the non-zero absolute value of the projection Λ of the orbital moment on the molecular axis is double-degenerate (the two levels differing only by the sign of the projection have the same energy), but the allowance for this coupling removes the degeneracy, causing the Λ-doubling. The radiative transitions from the Λ-doubling-caused pair of levels of the electronic state $^1\Pi$ to one of the levels of the electronic state $^1\Sigma^+$ were used in paper [7] as a supersensitive method for measuring a quasistatic electric field in plasmas. So, our new diagnostic method can be also implemented by considering radiative transitions in a diatomic molecule from the Λ-doubling-caused pair of levels -e and +f of the rotational state J+1, belonging to the electronic state $^1\Pi$, to the +e level of either the rotational state J or the rotational state J+2, belonging to the electronic state $^1\Sigma^+$.

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